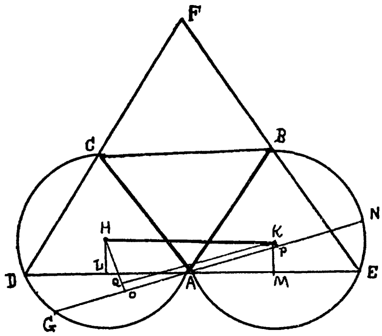
Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let ABC be the given triangle. On AC, AB describe segments to contain angles of 60°. Let H, K be the centers of these circles. Join HK and parallel to HK draw DAE. Join DC and EB, and produce DC, EB until they meet in F. Then FDE is the required maximum equilateral triangle.



$$\angle D = \angle E = \angle F = 60^\circ.$$

Draw any other line GN through A. Let fall the perpendiculars HL, KM, HO, KP, and draw KQ parallel to GN.

$$KQ = OP = \tfrac{1}{2}GN;\ HK = LM = \tfrac{1}{2}DE,\ HK > KQ. \quad \therefore DE > GN.$$

$\therefore DE$ is the longest line that can be drawn through A.

$\therefore FDE$ is the maximum equilateral triangle required.

The same method will serve to describe the maximum triangle having any angles, about ABC.

Also solved by J. Scheffer.

MISCELLANEOUS.

155. Proposed by A. H. HOLMES, Brunswick, Maine.

There are two vessels, one containing a gallons of alcohol, the other b gallons of water. Suppose that c gallons are simultaneously taken from each and poured into the other, how many times must this be done so that there will be the same proportion of alcohol to water in each vessel?

I. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let $F(x)$ represent the number of gallons in the first vessel after x operations, then the number of gallons in the first vessel after the next operation will be $F(x)-\frac{c}{a}F(x)+\frac{c}{b}[a-F(x)]$. $\therefore F(x+1)-[1-(\frac{c}{a}+\frac{c}{b})]F(x)=\frac{ac}{b}$.

By the calculus of finite differences, we get

$$F(x)=\frac{a^2}{a+b}+C[1-(\frac{c}{a}+\frac{c}{b})]^x.$$

Since for $x=0$, $F(x)=a$, we get $C=\frac{ab}{a+b}$.

$$\therefore F(x)=\frac{a^2}{a+b}+\frac{ab}{a+b}[1-(\frac{c}{a}+\frac{c}{b})]^x.$$

It is easily seen that when the same proportion of alcohol to water prevails, the contents of the alcohol in the first vessel will be$=a^2/(a+b)$.

$\therefore x$ must be$=\infty$.

II. Solution by G. B. M. ZERR. A. M., Ph. D.. Parsons, W. Va.

After the first operation, there are $a-c$ gallons of alcohol in the first vessel, and c gallons of alcohol in the second vessel. After the second operation, there are $a-2c+c^2(1/a+1/b)$ gallons of alcohol in the first vessel, and $2c-c^2(1/a+1/b)$ gallons of alcohol in the second vessel.

Let $A=c(1/a+1/b)$. Then, after the third operation, there are $a-3c+3Ac-A^2c$ gallons of alcohol in the first vessel, and $3c-3Ac+A^2c$ gallons of alcohol in the second vessel. After the nth operation there are

$$a-nc+\frac{n(n-1)}{2!}Ac-\frac{n(n-1)(n-2)}{3!}A^2c+\ldots\ldots\pm A^{n-1}c=a+\frac{c(1-A)^n-c}{A}$$ gallons of

alcohol, and $\dfrac{c-c(1-A)^n}{A}$ gallons of water in the first vessel, and

$$nc-\frac{n(n-1)}{2!}Ac+\frac{n(n-1)(n-2)}{3!}A^2c-\ldots\ldots\pm A^{n-1}c=\frac{c-c(1-A)^n}{A}$$ gallons of alcohol, and $b+\dfrac{c(1-A)^n-c}{A}$ gallons of water in the second vessel.

$$\therefore \frac{Aa+c(1-A)^n-c}{c-c(1-A)^n}=\frac{c-c(1-A)^n}{Ab+c(1-A)^n-c}.$$

$$\therefore (1-A)^n=\frac{c(a+b)-Aab}{c(a-b)}=0. \quad \therefore n=-\infty, \text{ or } A=1.$$

$\therefore$ The result stated can only happen when $a=b=2c$, then $n=1$.

156. Proposed by R. D. CARMICHAEL, Hartselle, Ala.

There exist no multiply perfect odd numbers of multiplicity n containing only n distinct primes.

Solution by JACOB WESTLUND, Ph. D., Purdue University, Lafayette, Ind,

If n denotes the multiplicity of a multiply perfect number $p_1^{a_1}p_2^{a_2}\ldots\ldots p_i^{a_i}$, where $p_1, p_2, \ldots\ldots$ are distinct primes, we have

$$n=\frac{p_1-\dfrac{1}{p_1^{a_1}}}{p_1-1}\cdot\frac{p_2-\dfrac{1}{p_2^{a_2}}}{p_2-1}\ldots\ldots, \text{ and hence } n<\frac{p_1}{p_1-1}.\frac{p_2}{p_2-1}\ldots\ldots\frac{p_i}{p_i-1}.$$

Now, if $p_1>2$, we have